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Application of the Karhunen-Loève Procedure for the Characterization of Human Faces

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Abstract—The exploitation of natural symmetries (mirror images) in a well-defined family of patterns (human faces) is discussed within the framework of the Karhunen-Loève expansion. This results in an extension of the data and imposes even and odd symmetry on the eigenfunctions of the covariance matrix, without increasing the complexity of the calculation. The resulting approximation of faces projected from outside of the data set onto this optimal basis is improved on average.

Index Terms—Data compression, data extension, face characterization, Karhunen-Loève expansion, symmetric eigenfunctions.

I. INTRODUCTION

There are many examples of families of patterns for which it is possible to obtain a useful systematic characterization. Often, the initial motivation might be no more than the intuitive notion that the family is low dimensional, that is, in some sense, any given member might be represented by a small number of parameters. Possible candidates for such families of patterns are abundant both in nature and in the literature. Such examples include turbulent flows [1], human speech [2], and the subject of this correspondence, human faces. While the techniques applied in this investigation are well known, we show how natural symmetries of the pattern family may be exploited to obtain improvements in the method. Although the subject of this study is that of human faces, it might be used with advantage whenever there are natural symmetries in a family of patterns.

Current machine ability to process facial information falls far

short of natural human capacity to perform the task. Early efforts in computer face processing have generally taken feature-based approaches, e.g., [3]-[6]. A series of studies, summarized in [7], concerning the classification of facial profiles has a high success rate for relatively small data sets. A more global approach, based on the use of an "optimal linear autoassociative mapping," i.e., linear regression, has been used to recall images using degraded or rotated originals as stimuli [8]. A method known as WISARD (Wilkie, Aleksander, and Stonham's Recognition Device) based on neural network principles has also been applied to face recognition [9]. For a detailed review of the literature in computer face processing, see the recent paper by Bruce and Burton [10]. The emphasis of the current study, as in a previous study [11], is on providing a reduced parametrization, and consequent data reduction, for 2-D digital images of faces. Here, faces are represented by the appropriate superposition of macrofeatures which are objectively generated on a statistical basis. For further perspective on the methodology, there are studies relating this type of approach to the cognitive psychology of face processing [12], [13].

The treatment presented here is based on the Karhunen-Loève expansion [14]-[17], although it also goes by other names, e.g., principal component analysis [18] and the Hotelling Transform [19]. The idea seems to have been first proposed by Pearson in 1901 [20] and then again by Hotelling in 1933 [21]. The method was introduced into the realm of pattern recognition by Watanabe in 1965 [2]. The goal of the approach is to represent a picture of a face in terms of an optimal coordinate system. Among the optimality properties is the fact that the mean-square error introduced by truncating the expansion is a minimum. The set of basis vectors which make up this coordinate system will be referred to as eigenpictures. They are simply the eigenfunctions of the covariance matrix of the ensemble of faces.

Rather than apply this procedure directly, we first extend the ensemble by including reflections about a midline of the faces, i.e., the mirror imaged faces. Using this extended ensemble in the computation of the covariance matrix imposes even and odd symmetry on the eigenfunctions. There is no cost in this modification because we are not actually doubling the size of the matrix in the eigenvector calculation. As shown in Section III, the symmetry allows the problem to be decoupled into two problems, each having the same complexity as the problem for the unextended ensemble. As a consequence of this procedure, the approximation error for pictures not included in the extended ensemble is reduced.

Although we make no attempt to relate the analytical techniques to human methods for face processing, we offer the following speculation. There is considerable evidence to indicate that the brain processes information along parallel pathways. See, for example, the paper by Anderson and Hinton for a discussion and review of the neurophysiological evidence [22]. Thus, it is natural to propose that an individual recognition task might be built up of several parallel tasks. For instance, we can imagine the eyes, nose, mouth, and ears being analyzed in parallel. This might explain why we describe an individual, for example, as having another person's eyes. The procedure described in this paper lends itself naturally to such a parallel approach. Clearly, for these subportions, the individual rates of convergence will be faster than for the face taken as a whole.

Previously we considered a cropping portion of a picture containing only the eyes and nose [11]. In the current investigation, we look at a cameo of the full face containing the eyes, nose, and mouth. As an evaluation of the success of the procedure, we project faces from outside of the data set onto the set of optimal basis vectors. As discussed in Section VII, this estimate is an upper bound for the error, on average. The error, averaged over ten faces, for a 50-term approximation was 3.68 percent. The success that this small error indicates is supported by the subjective evaluation provided by the human eye.

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II. FORMULATION

A picture of a face is represented by a scalar function $\varphi(x)$ of position $x = (x, y)$, with the picture centered on the midline $x = 0$. In actuality, we have a digital photograph consisting of a matrix φ of integral intensities or grey levels φ_{ij} ranging from 0 to 255. We consider an extended ensemble of pictures $\{\varphi^{(n)}(x, y)\} \cup \{\varphi^{(n)}(-x, y)\}$, $n = 1, 2, \dots, M$.

The average or composite face is then given by

$$\bar{\varphi} = \langle \varphi \rangle = \frac{1}{2M} \sum_{n=1}^M (\varphi^{(n)}(x, y) + \varphi^{(n)}(-x, y)). \quad (1)$$

We will say that a picture is even (in the midline) if

$$\varphi(x, y) = \varphi(-x, y). \quad (2)$$

and odd if

$$\varphi(x, y) = -\varphi(-x, y). \quad (3)$$

In keeping with customary practice, we focus on deviations from the mean face since this leads to a more efficient approach. Specifically, we form a new ensemble of mean subtracted faces:

$$\phi^{(n)} = \varphi^{(n)} - \bar{\varphi} \quad (4)$$

which we will refer to as caricatures. The average face and a typical mean subtracted picture are shown in Plate 1.

III. ANALYTICAL METHODS

By taking a distinguishable digital picture of a human face, we are, in fact, determining an upper bound on the dimensionality of the set of all human faces, namely, the number of pixels in the picture. We have found that 128×128 pixels, $O(10^4)$, gives a reasonable likeness, but as an estimate on dimension, this is crude. By continuously reducing the spatial resolution of the pictures while retaining recognizability, one could improve somewhat on this estimate [23].

It is reasonable to conjecture that the dimensionality of the set of human faces will be fairly small. Humans, after all, are prototypical face recognizers, and do so with such amazing speed that we might conclude that the quantity of information being processed is small, possibly in addition to being processed in parallel.

It seems apparent that the most natural coordinate system for our task will be data dependent. Intuitively, the basis vectors should in some sense be representative of the members of the ensemble. Such a coordinate system, also processing a host of optimality properties, is provided by the Karhunen-Loève expansion where the eigenfunctions are, in fact, admixtures of the ensemble. Hence, our basis will consist of the eigenfunctions of the integral equation

$$\int \mathcal{C}(x, x') u(x') dx' = \lambda u(x) \quad (5)$$

where the kernel is given by

$$\mathcal{C}(x, y, x', y') = \frac{1}{2M} \sum_{n=1}^M (\phi^{(n)}(x, y) \phi^{(n)}(x', y') + \phi^{(n)}(-x, y) \phi^{(n)}(-x', y')). \quad (6)$$

Within this framework, the coefficients in the expansion are uncorrelated, and each eigenvalue represents the statistical variance of the corresponding coefficient in the expansion, a property we will use when evaluating the results of the transformation. As is directly verified, we can rewrite \mathcal{C} as the sum of an even kernel \mathcal{C}_e and an odd kernel \mathcal{C}_o :

$$\mathcal{C}(x, x') = \mathcal{C}_e(x, x') + \mathcal{C}_o(x, x') \quad (7)$$

where

$$\mathcal{C}_e = \frac{1}{4M} \sum_{n=1}^M (\phi^{(n)}(x, y) + \phi^{(n)}(-x, y)) (\phi^{(n)}(x', y') + \phi^{(n)}(-x', y')) \quad (8)$$

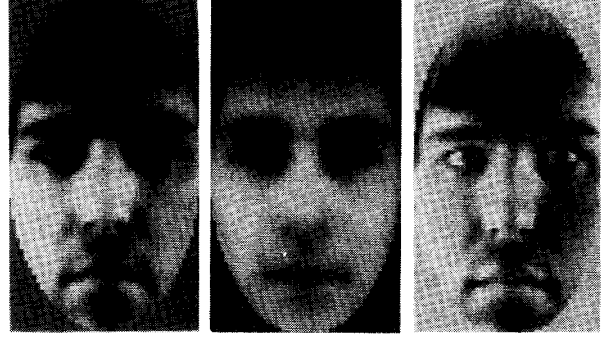


Plate 1. From left to right: sample face, average taken over extended ensemble, caricature of sample face.

and

$$\mathcal{C}_o = \frac{1}{4M} \sum_{n=1}^M (\phi^{(n)}(x, y) - \phi^{(n)}(-x, y)) (\phi^{(n)}(x', y') - \phi^{(n)}(-x', y')). \quad (9)$$

We remark that the kernels \mathcal{C}_e and \mathcal{C}_o are orthogonal and that their eigenvalues are even and odd, respectively. The eigenvectors of \mathcal{C}_e belong to the nullspace of \mathcal{C}_o and vice versa. Also, if $u \in E(\mathcal{C}_e)$ and $v \in E(\mathcal{C}_o)$, then $u + v \in E(\mathcal{C}_e + \mathcal{C}_o) = E(\mathcal{C})$ where $E(\mathcal{C})$ represents the eigenspace of the kernel \mathcal{C} . In other words, $E(\mathcal{C})$ can be expressed as the direct sum of $E(\mathcal{C}_e)$ and $E(\mathcal{C}_o)$, i.e.,

$$E(\mathcal{C}) = E(\mathcal{C}_e) \oplus E(\mathcal{C}_o). \quad (10)$$

If we define

$$\alpha^{(k)}(x, y) = \phi^{(k)}(x, y) + \phi^{(k)}(-x, y) \quad (11)$$

and

$$\beta^{(k)}(x, y) = \phi^{(k)}(x, y) - \phi^{(k)}(-x, y). \quad (12)$$

then it follows that we should consider the following two decoupled problems:

$$\int \mathcal{C}_o u_o dy' dx' = \lambda u_o \quad (13)$$

$$\int \mathcal{C}_e u_e dy' dx' = \lambda u_e \quad (14)$$

where

$$\mathcal{C}_e = \frac{1}{4M} \sum_{n=1}^M \alpha^{(n)}(x, y) \alpha^{(n)}(x', y') \quad (15)$$

and

$$\mathcal{C}_o = \frac{1}{4M} \sum_{n=1}^M \beta^{(n)}(x, y) \beta^{(n)}(x', y'). \quad (16)$$

We can view these two problems as equivalent to starting out with two separate ensembles $\{\alpha^{(n)}\}$ and $\{\beta^{(n)}\}$, $k = 1, 2, \dots, M$ consisting of even and odd pictures, and then proceeding with the two cases independently. We have shown that the eigenfunctions of (13) and (14) taken together form the solution set of (5).

The discrete formulation of the problem (13) [or (14)] involves computing the eigenvectors of the equation

$$\mathcal{C}u^{(n)} = \lambda^{(n)}u^{(n)} \quad (17)$$

where \mathcal{C} is now taken to be the discrete version of (15) [or (16)], i.e., it is a symmetric, nonnegative matrix. Alternatively, we can consider the equivalent variational formulation of the above problem. To determine the k th eigenvector of (17), we choose $u^{(k)}$ such

that

$$\lambda_k = \frac{1}{M} \sum_{n=1}^M (\mathbf{u}^{(k)}, \alpha^{(n)})^2 \quad (18)$$

is a maximum subject to the side constraints

$$(\mathbf{u}^{(k)}, \mathbf{u}^{(n)}) = \delta_{kl}, \quad l \leq k \quad (19)$$

with the usual Euclidian inner product. To integrate this, we observe that, on average, the members of $\{\alpha^{(n)}\}$ have their greatest component in the direction $\mathbf{u}^{(1)}$.

Since the kernels C_o and C_e are degenerate, we can represent the solutions by

$$u_o = \sum_k b_k \beta^{(k)}(x, y) \quad (20)$$

$$u_e = \sum_k a_k \alpha^{(k)}(x, y) \quad (21)$$

which, on substitution into (13) and (14), yields the two reduced problems

$$\sum_m L_{mn} a_m = \lambda a_n \quad (22)$$

$$\sum_m P_{mn} b_m = \lambda b_n \quad (23)$$

where $L_{mn} = (\alpha^m, \alpha^n)$ and $P_{mn} = (\beta^m, \beta^n)$. Thus, we see that the limiting factor in the calculation is that we must solve the eigenvector problem for an $M \times M$ matrix where M is the size of the unextended ensemble. However, there are techniques to deal with ensemble sizes that are too large to be done in one pass; for instance, see *the method of partitions* [24]. Note that the resolution of the pictures is not a real constraint since it only enters into the problem in the form of add/multiplies. Of course, too much resolution may present a practical problem.

IV. DATA ACQUISITION

The faces used in this experiment are the same as in the previous investigation. We use 100 pictures in the ensemble (or 200 in the extended ensemble), keeping some ten pictures aside for reasons to be discussed later. Each picture was captured and digitized individually using an IVS-100 image processor. Faces were lined up using a cross-hair overlay displayed on a video monitor. The vertical line passed through the symmetry line of the face and the horizontal line through the eyes. The field depth was also adjusted to make the facial width for each picture the same. In order to maximize the alignment, the pictures were later scaled and translated to fit a template which fixed the interocular distance. However, in most cases, the corrections were negligible. See the picture on left of Plate 1 for a representative member of the ensemble.

The pictures were taken under background lighting conditions, which varied with the time of day. Errors introduced in this way were partially corrected by employing the following normalization scheme. A picture can be equivalently viewed as an array of reflectivities $r(x)$. Thus, under a uniform illumination I , the corresponding picture is given by

$$\tilde{\varphi}(x) = Ir(x) \quad (24)$$

The normalization comes in imposing a fixed level of illumination I_0 at a reference point x_0 on a picture. The *normalized* picture is given by

$$\varphi(x) = I_0 \tilde{\varphi}(x) / \tilde{\varphi}(x_0). \quad (25)$$

In actual practice, we used the average of two reference points, one under each eye, each consisting of a 2×2 array of pixels.

The ensemble was deliberately chosen to be homogeneous, i.e., it consists of Caucasian males with no facial hair and eyeglasses removed. Otherwise, it is a fairly random selection of Brown University students and faculty who were passing through the Engineering Building, possibly a little too slowly. In the present investigation, we consider an oval-shaped portion of the face containing

essentially the eyes, nose, and mouth. The oval picture fits into a square of dimension 91×51 . We eliminated most of the hair as it significantly reduced the accuracy of the expressions. In any case, it would be possible to carry out a similar procedure on the complement of the portion of the face that was kept, and then fit the two together later.

V. RESULT

In Plate 2, the first nine eigenpictures (ordered by the size of their corresponding eigenvalues, starting with the largest, left to right and top to bottom) are shown. They are displayed by mapping the computed values to integers in the interval $[0, 255]$. The background has a fixed grey scale value of 128 and represents the zero level of the eigenpicture. Portions lighter than this are positive; portions darker are negative. Note also that the distinction between positive and negative is somewhat arbitrary since it is always permissible to multiply an eigenvector by a scalar, e.g., -1 .

It is possible to view the method as extracting facial features, at least in a statistical sense. For example, the first eigenpicture has a large extremum on the forehead, which is a direct result of the wide variation in the amount of hair present in the cropped pictures. Analogous statements are clearly possible for the other eigenpictures.

It is also interesting to consider the distribution of the eigenvalues corresponding to even and odd eigenpictures. Not surprisingly, the majority of the eigenpictures (five of the first six) corresponding to the largest eigenvalues are even. In fact, it might be regarded as surprising that the third eigenpicture is odd in view of the basic symmetrical nature of a human face. This result is very probably due to asymmetries in the background lighting that occurred during the picture acquisitions. Examining the coefficients of the eigenpictures for any given face will give a relative measure of its symmetry. For example, if there is a relatively large coefficient corresponding to eigenpicture number three, chances are the face is more asymmetrical than average.

It is interesting to compare the similarities and differences of the first several eigenpictures for the case with imposed symmetry to the case with the unextended ensemble. In Plate 3, the first five eigenpictures for both the extended (top row) and unextended (bottom row) ensembles are shown. They are displayed in black (positive) and white (negative) to emphasize their symmetry; however, this does lose the amplitude information apparent in Plate 2. We see remarkable similarities between the two sets. Most striking is that the eigenpictures of the original ensemble have nearly even and odd symmetries (compare the third eigenpicture in the top row to the fifth eigenpicture in the bottom row). The modification of extending the data through symmetry considerations might be thought of as directing the method where it is already heading. In other words, it could be viewed as an acceleration of convergence.

VI. EIGENPICTURE RECONSTRUCTION

Any picture in the ensemble can be represented exactly as the sum of the eigenpictures. Specifically, for any member of the population, we can write

$$\varphi = \bar{\varphi} + \sum_{n=1}^{2M} a_n \mathbf{u}^{(n)} \quad (26)$$

where

$$a_n = (\mathbf{u}^{(n)}, \varphi - \bar{\varphi}). \quad (27)$$

We next look at how much error is introduced by truncating this series, i.e., we consider the approximation

$$\varphi \approx \bar{\varphi} + \Phi^N = \varphi^N \quad (28)$$

where

$$\Phi^N = \sum_{n=1}^N a_n \mathbf{u}^{(n)}. \quad (29)$$

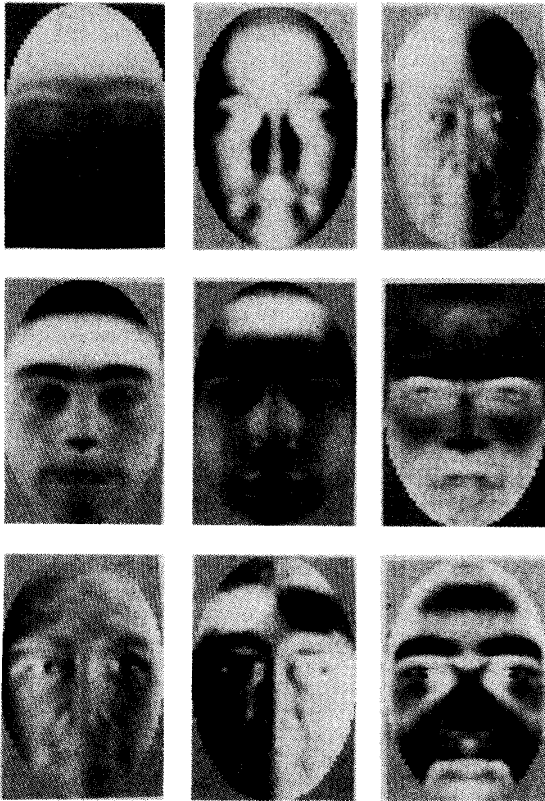


Plate 2. First nine eigenpictures, in order from left to right and top to bottom.

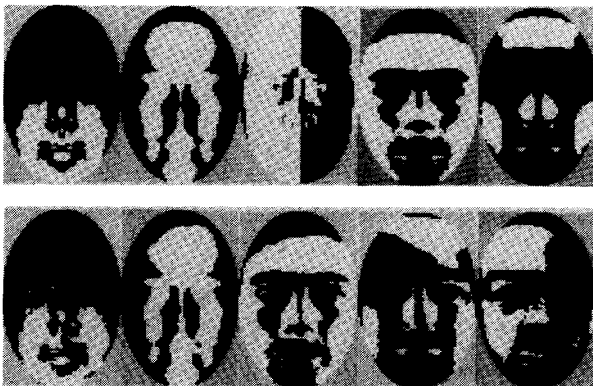


Plate 3. Binary mapped eigenpictures for both extended (top row) and unextended (bottom row) ensembles.

One can quantify the error of the approximation as

$$E_N = \|\varphi - \varphi^N\| / \|\varphi\| \quad (30)$$

where the norm is defined by

$$\|\varphi\| = (\varphi, \varphi)^{1/2}.$$

It measures the magnitude of the error vector, normalized by the vector representing the face that is being reconstructed. This measure is not supposed to be equivalent to the *human eye* in determining the quality of a reconstruction. However, the goal is still to form a *recognizable* reconstruction. See Plate 4 for pictures of

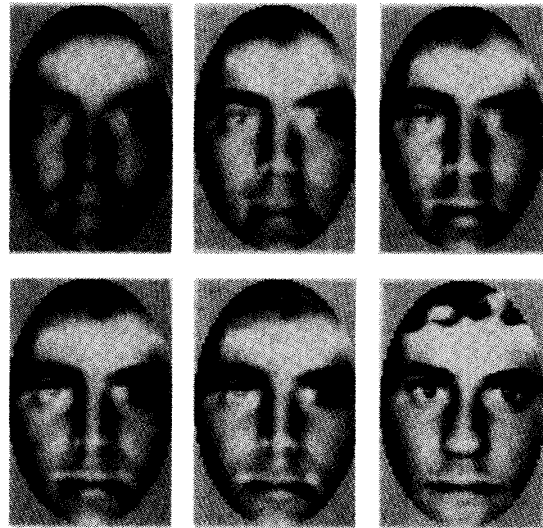


Plate 4. Approximation to the exact picture of caricature (lower right corner) using 10, 20, 30, 40, 50 eigenpictures. The original picture is not a member of the ensemble.

the reconstructions of the caricature of a typical face outside of the ensemble for $N = 10, 20, 30, 40, 50$. In Plate 5, two more typical approximates of data from outside of the original extended ensemble are shown. The approximation for $N = 50$ (left) is compared to the exact picture (right) in each case.

The convergence error E_N , plotted versus N , for the approximation shown in Plate 4 is given by the solid line in Fig. 1. The dashed curve represents the error averaged over a set of ten faces chosen at random from outside of the ensemble. In Fig. 2, we compare the errors, again averaged over ten faces projected from outside the data set, for the approximations using the symmetrical basis (lower curve) and the nonsymmetrical basis. At $N = 50$, the extended basis gives an error of 3.68 percent compared to 3.86 percent for the unextended set.

In addition, we compute the fraction of the total variance contained in the first N terms q_N as a function of N where

$$q_N = \frac{\sum_{i=1}^N \lambda^{(i)}}{\sum_{i=1}^{2M} \lambda^{(i)}}.$$

The first ten terms contain 82 percent of the variance, while by $N = 50$, we are up to 95 percent (see Fig. 3). We also plot $\lambda^{(n)}/\lambda_{\max}$ versus n in Fig. 4. Here, we see that the global Karhunen-Loève estimate of the dimensionality (the value of the index i for which $\lambda^{(i)}/\lambda_{\max} = 0.01$) of the set is about 21.

VII. ERROR ESTIMATION

One must exercise some care in making statements about the error of the approximation. Within the framework of finite dimensional vector spaces, it is possible to determine meaningful upper bounds on the error estimate of our truncated expansion even if our ensemble of pictures is too small. We begin by assuming that V is a finite dimensional vector space which contains all human faces. It is reasonable to assume that the dimension of V , say N , is finite in view of our earlier remarks (see "Results"). However, we do not restrict the total number of faces M to be finite. We will consider an example to be *too small* if it does not span V . Let the space spanned by an ensemble of size M , V_M , have dimension $D_M \leq N$. If our ensemble is too small, then estimates for both the accuracy of the approximation for a member of the ensemble and the total

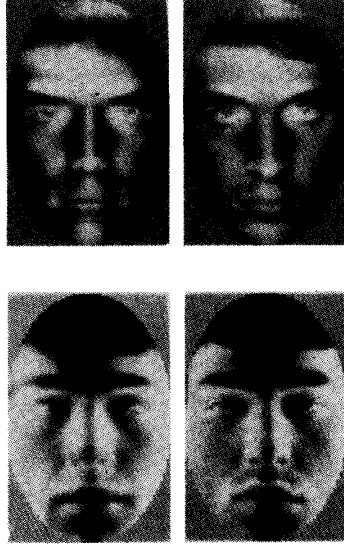


Plate 5. Fifty-term approximations of two sample caricatures taken from outside of the extended ensemble. In each case, the exact caricature is to the right of the approximation.

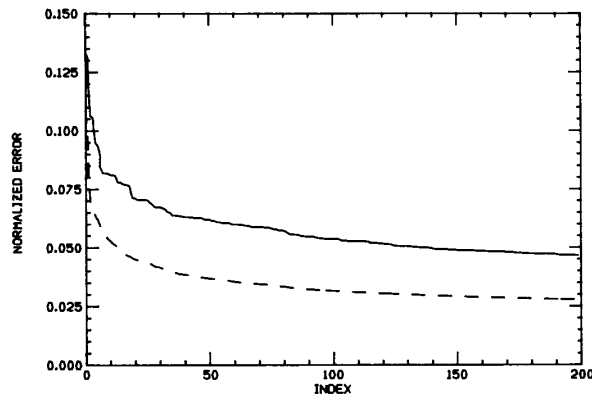


Fig. 1. E_N versus N for approximation shown in Plate 4 shown as continuous curve. Dashed curve corresponds to the error averaged over ten faces taken from outside of the ensemble.

variance contained in the first k terms are going to be too optimistic. Clearly, if we choose $M \ll N$, the errors for members of the ensemble will have no meaning, even though they look very attractive. However, if we consider elements of V not wholly contained in V_M , we can make some meaningful statements.

Let $\psi \in V$, but $\psi \notin V_m$, and let $\phi \in V_M$. Also, let P_k^M be the orthogonal projection onto V_k , $k \leq M$. Define

$$e_1(M; k) = E_{c_{V_M}} \|\psi - P_k^M \psi\|,$$

$$e_2(M; k) = E_{V_M} \|\phi - P_k^M \phi\|$$

where E_{V_M} denotes expected value over the set V_M , and c_{V_M} is the space spanned by members of the ensemble in V , but not in V_M . We state the following theorem without proof.

Theorem: 1) For any fixed $k \leq M$, $e_1(M; k)$ decreases monotonically to a constant $l_1(k)$ as M increases, 2) similarly, $e_2(M; k)$ increases monotonically to $l_2(k)$, 3) $l_1(k) = l_2(k)$.

The above theorem has a useful interpretation. Namely, we can obtain an upper bound for the average error even if our ensemble size is too small. In other words, on average, we will do no worse

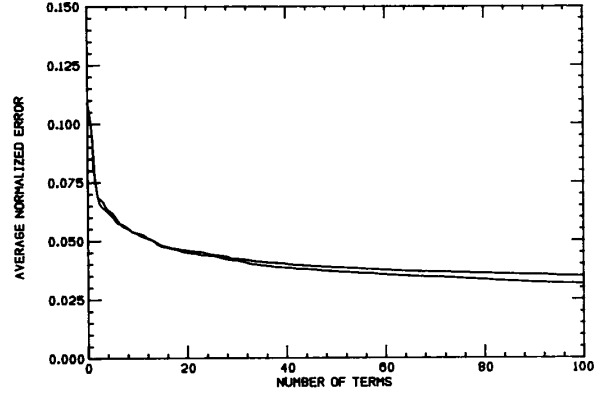


Fig. 2. Convergence error comparison (averaged over ten faces from outside of ensemble) versus number of eigenpictures used in the expansion for the symmetric basis and nonsymmetric basis (lower curve).

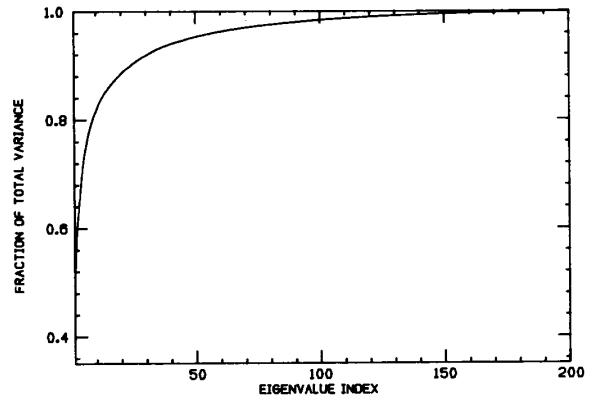


Fig. 3. Fraction of total variance q_N versus number of terms N in expansion.

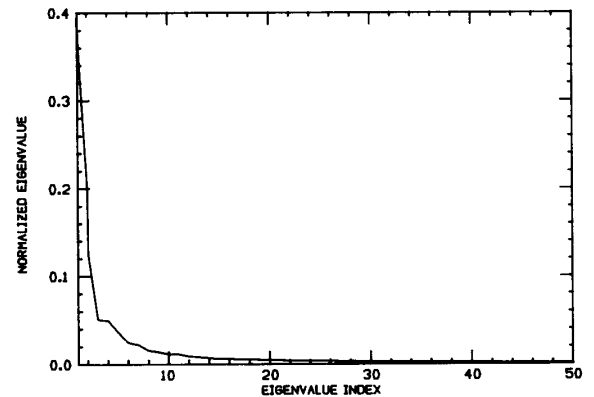


Fig. 4. Eigenvalues normalized by the maximum eigenvalue versus index.

than our upper error estimate found by projecting faces from outside of the ensemble. Also, as the ensemble size increases, the error will improve, up to the point where M is large enough.

VIII. DISCUSSION

Part of our discussion has centered around the notion of data extension using the natural symmetry of a pattern. We showed why

the resulting eigenpictures are necessarily even and odd. Patterns are now represented in terms of a basis possessing more structure, thus providing further characterization. Also, in hindsight, we can see that the eigenpictures corresponding to the unextended ensemble are nearly even and odd as well, a rather surprising result. In light of this fact and the improved approximations, we view the modification as beneficial.

Another theme of this correspondence has been data compression. We see that we can replace a 91×51 array by a 50-term expansion and retain a reasonable likeness, roughly a 100:1 compression ratio. This number of terms should decrease further still given a larger ensemble size in view of the theorem in Section VII. This conclusion is drawn from the fact that members of the ensemble have more accurate expansions than projections from outside of the ensemble.

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Correction to "Image Computations on Meshes with Multiple Broadcast"

Due to a very unfortunate compositor's error not noticed by IEEE Staff, the name of the first author of the paper, "Image Computations on Meshes with Multiple Broadcast,"¹ was misspelled in the biography which was sent in a separate mailing. The correct spelling is V. K. Prasanna-Kumar.

We sincerely regret this error.

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