

MVA "Kernel methods"

Homework 5

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Exercise 1.

The convolution between two functions $f, g : \mathbb{R} \rightarrow \mathbb{R}$ is defined by:

$$f \star g(x) = \int_{-\infty}^{\infty} f(u)g(x-u)du,$$

when this integral exists.

Let now the function:

$$I(x) = \begin{cases} 1 & \text{si } -1 \leq x \leq 1, \\ 0 & \text{si } x < -1 \text{ ou } x > 1, \end{cases}$$

and $B_n = I^{\star n}$ for $n \in \mathbb{N}_*$ (that is, the function I convolved n times with itself: $B_1 = I, B_2 = I \star I, B_3 = I \star I \star I$, etc...).

Is the function $k(x, y) = B_n(x - y)$ a positive definite kernel over $\mathbb{R} \times \mathbb{R}$? If yes, describe the corresponding reproducing kernel Hilbert space.

Exercise 2.

Are the following functions positive definite kernels?

$$\forall x, y \in \mathbb{R}, \quad K_1(x, y) = \frac{1}{2 - e^{-\|x-y\|^2}}$$

$$\forall x, y \in \mathbb{R}, \quad K_2(x, y) = \max(0, 1 - |x - y|)$$

$$\forall x, y \in \mathbb{R}^+, \quad K_3(x, y) = \frac{1}{1 + x + y}$$

Exercise 3.

Describe the functions $\phi : [0, 1] \mapsto \mathbb{R}$ such that:

$$K(x, y) = \phi(\max(x + y - 1, 0))$$

is a positive definite kernel on $[0, 1]$.

Exercise 4.

Describe the functions $\phi : \mathbb{R}^+ \mapsto \mathbb{R}$ such that:

$$K(x, y) = \phi(\max(x, y))$$

is a positive definite kernel on \mathbb{R}^+ .