Homework 3

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1 B_n-splines

The convolution between two functions $f, g : \mathbb{R} \to \mathbb{R}$ is defined by:

$$f \star g(x) = \int_{-\infty}^{\infty} f(u)g(x-u)du,$$

when this integral exists.

Let now the function:

$$I(x) = \begin{cases} 1 & \text{si } -1 \leq x \leq 1, \\ 0 & \text{si } x < -1 \text{ ou } x > 1, \end{cases}$$

and $B_n = I^{\star n}$ for $n \in \mathbb{N}_*$ (that is, the function I convolved n times with itself: $B_1 = I, B_2 = I \star I, B_3 = I \star I \star I$, etc...).

Is the function $k(x, y) = B_n(x - y)$ a positive definite kernel over $\mathbb{R} \times \mathbb{R}$? If yes, describe the corresponding reproducing kernel Hilbert space.

2 Kernel for events

Let $(\Omega, \mathcal{A}, P)$ be a probability space. Is the function

$$K : \mathcal{A} \times \mathcal{A} \to \mathbb{R}$$

defined by:

$$\forall (A, B) \in \mathcal{A}^2, \quad K(A, B) = P(A \cap B) - P(A)P(B)$$

a p.d. kernel?

3 More kernels...

Are the following functions positive definite kernels?

$$\forall x, y > 0, \quad K_1(x, y) = \frac{\min(x, y)}{\max(x, y)}$$

$$\forall x, y \in \mathbb{R}, \quad K_2(x, y) = \frac{1}{2 - e^{-\| x-y \|^2}}$$

$$\forall x, y \in \mathbb{R}, \quad K_3(x, y) = \max(0, 1 - |x - y|)$$