Inferring and using biological networks

Jean-Philippe Vert
Jean-Philippe.Vert@mines-paristech.fr
Mines ParisTech / Institut Curie / INSERM U900

Human Genome Center, Institute of Medical Science, University of Tokyo, August 4, 2009.
We have many genes and proteins..
Network 1: protein-protein interaction
Network 2: metabolic network
Network 3: gene transcriptional regulatory network
Biologists have collected a lot of data about proteins. e.g.,

- Gene expression measurements
- Phylogenetic profiles
- Location of proteins/enzymes in the cell
Problem 1: how to infer relationships between genes from biological data?
Problem 2: how to use biological networks to help in the analysis of genomic data?
1. How to infer relationships between genes from biological data?

2. How to use biological networks to help in the analysis of genomic data?
1. How to infer relationships between genes from biological data?

2. How to use biological networks to help in the analysis of genomic data?
Typical reverse engineering strategies

- Fit a dynamical system to time series (e.g., PDE, boolean networks, state-space models)
- Detect statistical conditional independence or dependency (Bayesian network, mutual information networks, co-expression networks, ...)

![Diagram showing different types of networks and inference process]
The known metabolic network of the yeast involves **769 proteins**.

Predict edges from distances between a variety of genomic data (expression, localization, phylogenetic profiles, interactions).
Large-Scale Mapping and Validation of *Escherichia coli* Transcriptional Regulation from a Compendium of Expression Profiles

Jeremiah J. Faith\(^1\text{a}\), Boris Hayete\(^1\text{a}\), Joshua T. Thaden\(^2\text{,}3\), Ilaria Mogno\(^2\text{,}4\), Jamey Wierzbowski\(^2\text{,}5\), Guillaume Cottarel\(^2\text{,}5\), Simon Kasir\(^1\text{,}2\), James J. Collins\(^1\text{,}2\), Timothy S. Gardner\(^1\text{,}2\)
Change of paradigm

Motivation
In actual applications,
- we know in advance parts of the network to be inferred
- the problem is to add/remove nodes and edges using genomic data as side information

Supervised method
- Given genomic data and the currently known network...
- Infer missing edges between current nodes and additional nodes.
Interlude: Pattern recognition

Given a training set of patterns in two classes, learn to discriminate them.

- Many algorithms (ANN, SVM, Decision trees, ...)

- Diagram showing two sets of points, one red and one yellow, indicating the separation of patterns in two classes.
Given a training set of patterns in two classes, learn to discriminate them

Many algorithms (ANN, SVM, Decision tress, ...)

Jean-Philippe Vert (ParisTech)

Inferring and using biological networks
Interlude: Pattern recognition

Given a training set of patterns in two classes, learn to discriminate them.

Many algorithms (ANN, SVM, Decision trees, ...)

Jean-Philippe Vert (ParisTech)
Given a training set of patterns in two classes, learn to discriminate them.

Many algorithms (ANN, SVM, Decision tress, ...)

Interlude: Pattern recognition
### Pattern recognition

Associate a binary label $Y$ to each data $X$.

### Graph inference

Associate a binary label $Y$ to each pair of data $(X_1, X_2)$.

### Two solutions

- Consider each pair $(X_1, X_2)$ as a single data -> learning over pairs
- Reformulate the graph inference problem as a pattern recognition problem at the level of individual vertices -> local models
### Pattern recognition

Associate a binary label $Y$ to each data $X$

### Graph inference

Associate a binary label $Y$ to each pair of data $(X_1, X_2)$

### Two solutions

- Consider each pair $(X_1, X_2)$ as a single data -> **learning over pairs**
- Reformulate the graph inference problem as a pattern recognition problem at the level of individual vertices -> **local models**
Pattern recognition for pairs

Formulation and basic issue

- A pair can be connected (1) or not connected (-1)
- From the known subgraph we can extract examples of connected and non-connected pairs
- However the genomic data characterize individual proteins; we need to work with pairs of proteins instead!

Known graph

Genomic data
Pattern recognition for pairs

Formulation and basic issue

- A pair can be connected (1) or not connected (-1)
- From the known subgraph we can extract examples of connected and non-connected pairs
- However the genomic data characterize individual proteins; we need to work with pairs of proteins instead!

![Known graph](image)

![Genomic data](image)
Pattern recognition for pairs

Formulation and basic issue

- A pair can be connected (1) or not connected (-1)
- From the known subgraph we can extract examples of connected and non-connected pairs
- However the genomic data characterize individual proteins; we need to work with pairs of proteins instead!
A simple idea is to \textit{concatenate} the vectors $u$ and $v$ to obtain a $2p$-dimensional vector of $(u, v)$:

$$\psi(u, v) = u \oplus v = \begin{pmatrix} u \\ v \end{pmatrix}. $$

\textbf{Problem:} a linear function then becomes additive...

$$f(u, v) = w^T \psi(u, v) = w_1^T u + w^T v.$$
A simple idea is to **concatenate** the vectors $u$ and $v$ to obtain a $2p$-dimensional vector of $(u, v)$:

$$\psi(u, v) = u \oplus v = \left( \begin{array}{c} u \\ v \end{array} \right).$$

**Problem**: a linear function then becomes **additive**...

$$f(u, v) = w^\top \psi(u, v) = w_1^\top u + w^\top v.$$
Other representations for pairs

Symmetric tensor product (Ben-Hur and Noble, 2006)

\[ \psi(u, v) = (u \otimes v) + (v \otimes u) . \]

**Intuition:** a pair \((A, B)\) is similar to a pair \((C, D)\) if:
- \(A\) is similar to \(C\) and \(B\) is similar to \(D\), or...
- \(A\) is similar to \(D\) and \(B\) is similar to \(C\).

Metric learning (V. et al, 2007)

\[ \psi(u, v) = (u - v) \otimes^2 . \]

**Intuition:** a pair \((A, B)\) is similar to a pair \((C, D)\) if:
- \(A - B\) is similar to \(C - D\), or...
- \(A - B\) is similar to \(D - C\).
Other representations for pairs

Symmetric tensor product (Ben-Hur and Noble, 2006)

\[ \psi(u, v) = (u \otimes v) + (v \otimes u) \, . \]

**Intuition:** a pair \((A, B)\) is similar to a pair \((C, D)\) if:
- \(A\) is similar to \(C\) and \(B\) is similar to \(D\), or...
- \(A\) is similar to \(D\) and \(B\) is similar to \(C\)

Metric learning (V. et al, 2007)

\[ \psi(u, v) = (u - v) \otimes^2 \, . \]

**Intuition:** a pair \((A, B)\) is similar to a pair \((C, D)\) if:
- \(A - B\) is similar to \(C - D\), or...
- \(A - B\) is similar to \(D - C\).
Supervised inference with local models

The idea (Bleakley et al., 2007)

- Motivation: define **specific models** for each target node to discriminate between its neighbors and the others
- Treat each node independently from the other. Then combine predictions for ranking candidate edges.
Supervised inference with local models

The idea (Bleakley et al., 2007)

- Motivation: define specific models for each target node to discriminate between its neighbors and the others
- Treat each node independently from the other. Then combine predictions for ranking candidate edges.
The LOCAL model
The LOCAL model
The LOCAL model
The LOCAL model
The LOCAL model
The LOCAL model
The LOCAL model
The LOCAL model

![Diagram of the LOCAL model with nodes and edges labeled with +1 and -1.]
The LOCAL model
The LOCAL model
The LOCAL model
The LOCAL model
The LOCAL model
The LOCAL model
The LOCAL model

[Diagram showing a network of nodes and connections]
A few remarks about the local approach

**Weak hypothesis:**
- if A is connected to B,
- if C is similar to B,
- then A is likely to be connected to C.

**Computationally:** much faster to train $N$ local models with $N$ training points each, than to train 1 model with $N^2$ training points.

**Caveats:**
- each local model may have very few training points
- no sharing of information between different local models
A few remarks about the local approach

- **Weak hypothesis:**
  - if A is connected to B,
  - if C is similar to B,
  - then A is likely to be connected to C.

- **Computationally:** much faster to train $N$ local models with $N$ training points each, than to train 1 model with $N^2$ training points.

- **Caveats:**
  - each local model may have very few training points
  - no sharing of information between different local models
A few remarks about the local approach

Weak hypothesis:
- if A is connected to B,
- if C is similar to B,
- then A is likely to be connected to C.

Computationally: much faster to train $N$ local models with $N$ training points each, than to train 1 model with $N^2$ training points.

Caveats:
- each local model may have very few training points
- no sharing of information between different local models
Results: protein-protein interaction (yeast)

(from Bleakley et al., 2007)
Results: metabolic gene network (yeast)

(from Bleakley et al., 2007)
Results: regulatory network (E. coli)

SIRENE = Supervised Inference of REgulatory NEtworks (Mordelet and V., 2008)

<table>
<thead>
<tr>
<th>Method</th>
<th>Recall at 60%</th>
<th>Recall at 80%</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIRENE</td>
<td>44.5%</td>
<td>17.6%</td>
</tr>
<tr>
<td>CLR</td>
<td>7.5%</td>
<td>5.5%</td>
</tr>
<tr>
<td>Relevance networks</td>
<td>4.7%</td>
<td>3.3%</td>
</tr>
<tr>
<td>ARACNe</td>
<td>1%</td>
<td>0%</td>
</tr>
<tr>
<td>Bayesian network</td>
<td>1%</td>
<td>0%</td>
</tr>
</tbody>
</table>
Prediction of missing enzyme genes in a bacterial metabolic network

Reconstruction of the lysine-degradation pathway of Pseudomonas aeruginosa

Yoshihiro Yamanishi, Hisaaki Mihara, Motoharu Osaki, Hisashi Muramatsu, Nobuyoshi Esaki, Tetsuya Sato, Yoshiyuki Hizukuri, Susumu Goto and Minoru Kanehisa

1 Bioinformatics Center, Institute for Chemical Research, Kyoto University, Japan
2 Division of Environmental Chemistry, Institute for Chemical Research, Kyoto University, Japan
3 Department of Biology, Graduate School of Science, Osaka University, Japan
Applications: missing enzyme prediction

LYSINE DEGRADATION

Penicillins and cephalosporins biosynthesis

2-Oxoadipate

S-Glutaryl-dihydrolipoamide

Crotonoyl-CoA

(S)-3-Hydroxybutanoyl-CoA

Acetoacetyl-CoA

Acetyl-CoA

Citrate cycle

1.2.4.2

2.6.1.39

2.1.1.20

2.1.1.60

2.1.1.35

6.2.1.6

1.3.99.7

1.2.1.13

6.2.1.48

1.5.1.10

1.2.1.1

1.6.1.21

1.5.99.3

1.5.1.9

1.5.1.18

1.5.1.16

1.5.1.17

1.5.1.30

1.5.1.19

1.5.1.17

1.5.1.19

1.5.1.19

N-Acetyllysine

N-Acetyllysine

Lysine biosynthesis

Biotin metabolism
Prediction of nitrogen metabolism-related genes in *Anabaena* by kernel-based network analysis

Shinobu Okamoto¹*, Yoshihiro Yamanishi¹, Shigeki Ehira², Shuichi Kawashima³, Koichiro Tonomura¹** and Minoru Kanehisa¹

¹ Bioinformatics Center, Institute for Chemical Research, Kyoto University, Uji, Japan  
² Department of Biochemistry and Molecular Biology, Faculty of Science, Saitama University, Saitama, Japan  
³ Human Genome Center, Institute of Medical Science, University of Tokyo, Meguro, Japan
Determination of the role of the bacterial peptidase PepF by statistical inference and further experimental validation

Liliana LOPEZ KLEINE\textsuperscript{1,2}, Alain TRUBUI\textsuperscript{1}, Véronique MONNET\textsuperscript{2}

\textsuperscript{1}Unité de Mathématiques et Informatiques Appliquées. INRA Jouy en Josas 78352, France.
\textsuperscript{2}Unité de Biochimie Bactérienne. INRA Jouy en Josas 78352, France.
Prediction at 60% precision, restricted to transcription factors (from Mordelet and V., 2008).
Outline

1. How to infer relationships between genes from biological data?

2. How to use biological networks to help in the analysis of genomic data?
Tissue classification from microarray data

Goal
- Design a classifier to automatically assign a class to future samples from their expression profile
- Interpret biologically the differences between the classes

Issue
20K+ genes but only <100 tumours
Linear classifiers and signatures

The model

- Each sample is represented by a vector $\mathbf{x} = (x_1, \ldots, x_p)$
- **Goal**: estimate a linear function:

  $$f_{\beta}(\mathbf{x}) = \sum_{i=1}^{p} \beta_i x_i + \beta_0.$$  

- **Interpretability**: the weight $\beta_i$ quantifies the influence of feature $i$ (but...)

\[ \text{Image of scattered data points with a linear decision boundary} \]
Linear classifiers

Training the model

- Minimize an empirical risk on the training samples:

  \[
  \min_{\beta \in \mathbb{R}^{p+1}} R_{\text{emp}}(\beta) = \frac{1}{n} \sum_{i=1}^{n} l(f_\beta(x_i), y_i),
  \]

- ... subject to some constraint on \( \beta \), e.g.:

  \[
  \Omega(\beta) \leq C.
  \]
Classical penalties

- **Feature selection** (NP-hard, many greedy variants exist):

  \[ \Omega_{\text{Best subset selection}}(\beta) = \| \beta \|_0 = \sum_{i=1}^{p} 1(\beta_i > 0). \]

- **Small weights** (SVM, ridge regression, ...):

  \[ \Omega_{\text{ridge}}(\beta) = \| \beta \|_2^2 = \sum_{i=1}^{p} \beta_i^2. \]

- **Sparsity-inducing convex priors** (computationnally tractable + feature selection):

  \[ \Omega_{\text{LASSO}}(\beta) = \| \beta \|_1 = \sum_{i=1}^{p} |\beta_i|. \]
Classical penalties

- **Feature selection** (NP-hard, many greedy variants exist):
  \[
  \Omega_{\text{Best subset selection}}(\beta) = \| \beta \|_0 = \sum_{i=1}^{p} 1(\beta_i > 0).
  \]

- **Small weights** (SVM, ridge regression, ...):
  \[
  \Omega_{\text{ridge}}(\beta) = \| \beta \|_2^2 = \sum_{i=1}^{p} \beta_i^2.
  \]

- **Sparsity-inducing convex priors** (computationally tractable + feature selection):
  \[
  \Omega_{\text{LASSO}}(\beta) = \| \beta \|_1 = \sum_{i=1}^{p} |\beta_i|.
  \]
Classical penalties

- **Feature selection** (NP-hard, many greedy variants exist):
  \[
  \Omega_{\text{Best subset selection}}(\beta) = \| \beta \|_0 = \sum_{i=1}^{p} 1(\beta_i > 0).
  \]

- **Small weights** (SVM, ridge regression, ...):
  \[
  \Omega_{\text{ridge}}(\beta) = \| \beta \|_2^2 = \sum_{i=1}^{p} \beta_i^2.
  \]

- **Sparsity-inducing convex priors** (computationnally tractable + feature selection):
  \[
  \Omega_{\text{LASSO}}(\beta) = \| \beta \|_1 = \sum_{i=1}^{p} | \beta_i |.
  \]
Why LASSO leads to sparse solutions

Geometric interpretation with $p = 2$
How protein networks can help us

- Basic biological functions usually involve the *coordinated action of several proteins*:
  - Formation of *protein complexes*
  - Activation of metabolic, signalling or regulatory *pathways*

- Many pathways and protein-protein interactions are *already known*

- **Hypothesis**: the signature should be “coherent” with respect to this *prior knowledge*
Hypothesis: adjacent genes should have similar weights in the signature.

Penalty function (Rapaport et al., 2007): 

\[ \Omega_{\text{smooth}}(\beta) = \sum_{i \sim j} (\beta_i - \beta_j)^2 \]
1. Use the gene network to extract the “important information” in gene expression profiles by Fourier analysis on the graph.

2. Learn a linear classifier on the smooth components with classical ridge penalty.
Illustration (yeast, high vs. low irradiation doses)

Fig. 5. ... work was supported by the grant ACI-IMPBIO-2004-47 of the French Ministry for Research and New Technologies.
Example: smooth and sparse signature

- **Hypothesis:**
  - the signature should be **sparse** (gene selection)
  - **connected genes** should have the same weight

- **Penalty function (Rapaport et al., 2008):**

  \[
  \Omega_{\text{piecewiseconstant}}(\beta) = \sum_{i \sim j} |\beta_i - \beta_j| + \lambda \sum_i |\beta_i|.
  \]

Geometric interpretation with \( p = 2 \)
Hypothesis:
- the signature should be **sparse** (gene selection)
- selected genes should form **dense connected components** (without any constraint of their relative weights)

Penalty function (Jacob et al., 2009):

\[
\Omega_{\text{intersection}}(\beta) = \sum_{i \sim j} \sqrt{\beta_i^2 + \beta_j^2},
\]

\[
\Omega_{\text{union}}(\beta) = \sup_{\alpha \in \mathbb{R}^p : \forall i \sim j, \|\alpha_i^2 + \alpha_j^2\| \leq 1} \alpha^\top \beta.
\]
Graph LASSO leads to structured sparsity

Groups \((1, 2)\) and \((2, 3)\). Left: \(\Omega_{\text{intersection}}(\beta)\). Right: \(\Omega_{\text{union}}(\beta)\). Vertical axis is \(\beta_2\).
Breast cancer data

- Gene expression data for 8,141 genes in 295 breast cancer tumors.
- Canonical pathways from MSigDB containing 639 groups of genes, 637 of which involve genes from our study.

<table>
<thead>
<tr>
<th>METHOD</th>
<th>$\ell_1$</th>
<th>$\Omega_{\text{OVERLAP}}^G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERROR</td>
<td>0.38 ± 0.04</td>
<td>0.36 ± 0.03</td>
</tr>
<tr>
<td># PATH.</td>
<td>148, 58, 183</td>
<td>6, 5, 78</td>
</tr>
<tr>
<td>PROP. PATH.</td>
<td>0.32, 0.14, 0.41</td>
<td>0.01, 0.01, 0.17</td>
</tr>
</tbody>
</table>

- Graph on the genes.

<table>
<thead>
<tr>
<th>METHOD</th>
<th>$\ell_1$</th>
<th>$\Omega_{\text{graph}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERROR</td>
<td>0.39 ± 0.04</td>
<td>0.36 ± 0.01</td>
</tr>
<tr>
<td>AV. SIZE C.C.</td>
<td>1.1, 1, 1.0</td>
<td>1.3, 1.4, 1.2</td>
</tr>
</tbody>
</table>
Conclusion

- A supervised machine learning formulation leads to promising results on the problem of inferring unknown relationships between genes and proteins.
- Conversely, biological networks can help fighting the curse of dimensionality for classification of high-dimensional genomic data.
- All this is progressing very quickly these days!
People I need to thank

- **Graph inference**: Yoshihiro Yamanishi, Minoru Kanehisa (Univ. Kyoto), Jian Qian, Bill Noble (Univ. Washington), Kevin Bleakley, Gerard Biau (Univ. Montpellier), Fantine Mordelet (ParisTech)

- **Using graphs**: Franck Rapaport, Emmanuel Barillot, Andrei Zinovyev (Institut Curie), Laurent Jacob (ParisTech), Guillaume Obozinski (Berkeley / INRIA)
Acknowledgement

This presentation is supported by a JSPS Invitation Fellowship Program for Research in Japan, hosted by Tatsuya Akutsu (Kyoto University)