Homework 1
Ms "Probability and Applications", Paris 6/7

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1 Kernel PCA

Let $S_{train} = (x_1, \ldots, x_n)$ and $S_{train} = (x_{n+1}, \ldots, x_{n+p})$ be two sets of points in a space endowed with a positive definite kernel K. Propose and implement an algorithm to project the set S_{test} onto the first principal directions obtained by kernel PCA on the set S_{train}

2 Kernel ridge regression with offset

Given a training set $(x_i, y_i)_{i=1,\ldots,n}$ where x_i is a point of a space endowed with a positive definite kernel K and y_i is a real number, the *kernel ridge regression with offset* algorithm solves:

$$\min_{f \in H_K, b \in \mathbb{R}} \frac{1}{n} \sum_{i=1}^{n} (f(x_i) + b - y_i)^2 + \lambda ||f||^2_{H_K} \,,$$

where H_K is the RKHS of the kernel K. Propose and implement an algorithm to find f and b.

3 Application

Download the data from the course web page. For a few kernels (e.g., linear and Gaussian with different bandwith):

- Visualize the training and testing sets by projecting them on the first 2 kernel principal directions computed on the training set.

- Train a kernel ridge regression model with offset on the training set, and compute the performance of the model on the training set and on the test set (using mean square error, MSE). Plot the training MSE and the testing MSE as a function of λ for the different kernels. Comment the results.